Orthophoto

Orthophoto

- Definition
  - an orthophoto is an image of the object surface in **orthogonal parallel Projection**
  - can be the surface of the earth, but also a building facade, a technical component ...
  - Orthophotos are usually generated synthetically (actually recording an orthogonal parallel projection is hard)
- Ingredients
  - an image of the surface of interest
  - interior and exterior orientation of the image
  - a digital model of the surface - for the earth, a digital surface model (DSM)
  - ***Note***: here only digital images are treated; before the advent of digital image processing there were analog machines for ortho-rectification

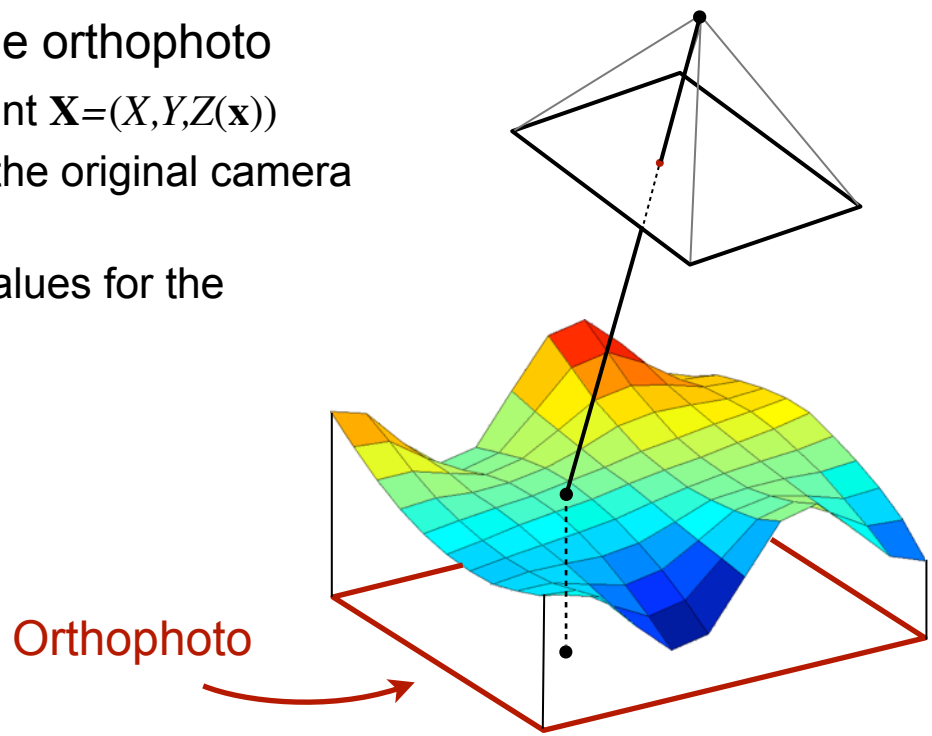
Orthophoto

- Example

Rectification

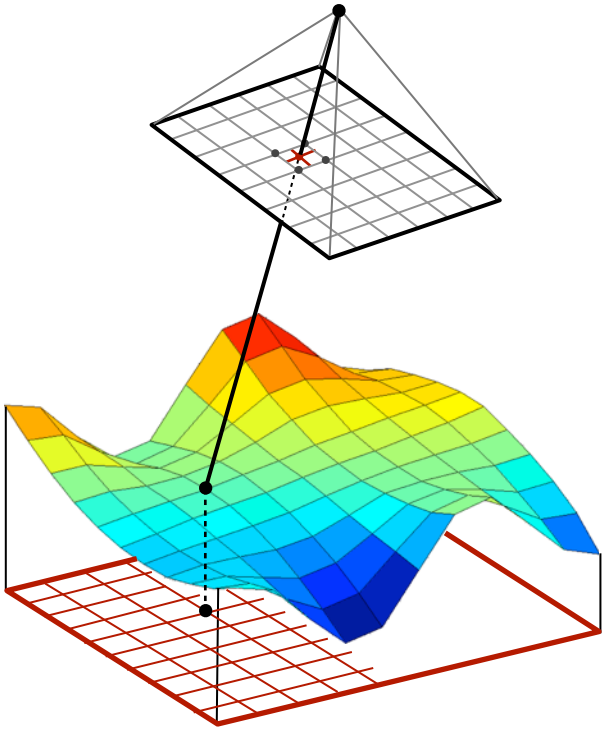
- Starting from the original image
  - project image to surface model
  - project textured surface model onto (X,Y)-plane

- Starting from orthophoto plane
  - for each point $\mathbf{x}_o=(X,Y)$ of the orthophoto
    - determine the surface point $\mathbf{X}=(X,Y,Z(\mathbf{x}))$
    - map the surface point to the original camera image $\mathbf{x}=\mathrm{P}\mathbf{X}$
    - read out the brightness values for the orthophoto: $I(\mathbf{x}_o) = I(\mathbf{x})$



Rectification

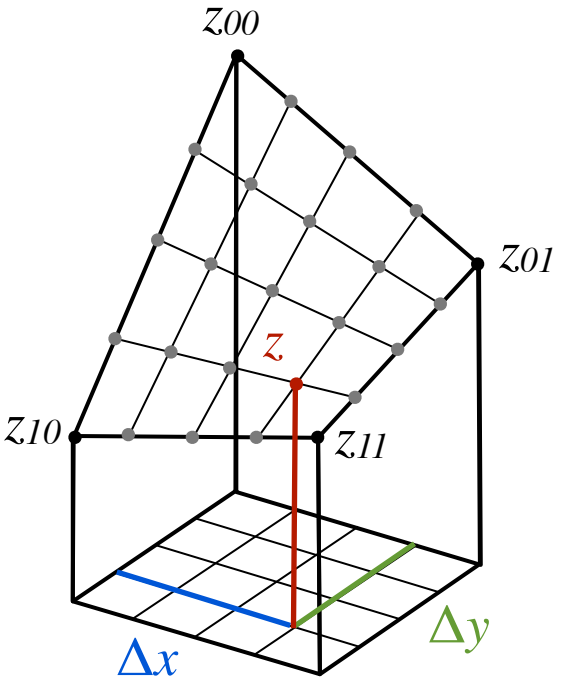
- in a discrete pixel raster
  - Define pixel positions in orthophoto
  - interpolate surface height at those positions
  - project 3D surface points to image points
  - interpolate brightness values for the orthophoto from the neighborhood in the original image



DTM Interpolation

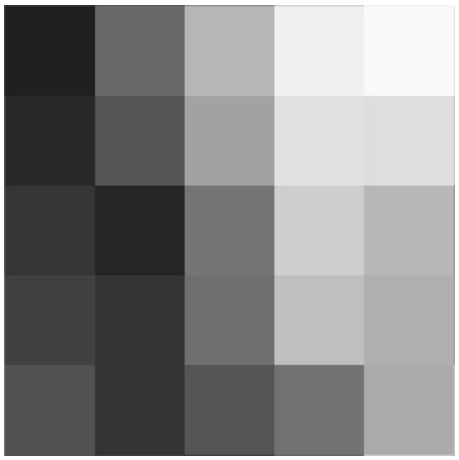
- Interpolation of surface (terrain) heights
  - at every position in the pixel raster of the orthophoto one needs to determine a height
- Example
  - Surface model in raster form (coarser than orthophoto)
  - bilinear interpolation

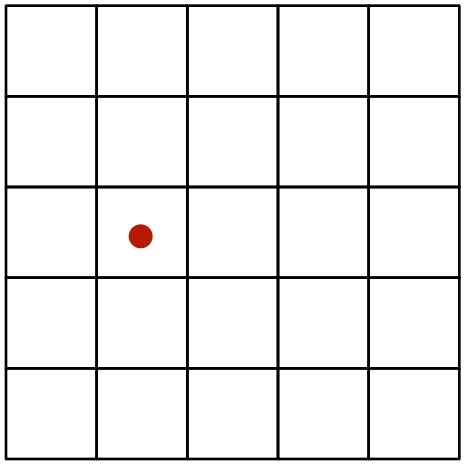
$$
\begin{aligned}
z \quad = \quad & (1-\Delta x) \cdot (1-\Delta y) \cdot z_{00} + \\
& \Delta x \cdot (1-\Delta y) \cdot z_{01} + \\
& (1-\Delta x) \cdot \Delta y \cdot z_{10} + \\
& \Delta x \cdot \Delta y \cdot z_{11}
\end{aligned}
$$



Resampling

- Estimation of brightness values I_{ortho} for the orthophoto
  - for each raster point $\mathbf{X}_{ortho}$ on the surface
    - project to camera image with collinearity $\mathbf{x}_{cam} = \mathrm{P}\ \mathbf{X}_{ortho}$
    - compute intensity $I(\mathbf{x}_{cam})$ from surrounding intensity values
    - invert projection, i.e. assign $I(\mathbf{x}_{ortho}) = I(\mathbf{x}_{cam})$
  - in multi-channel images: separately get intensity for each channel





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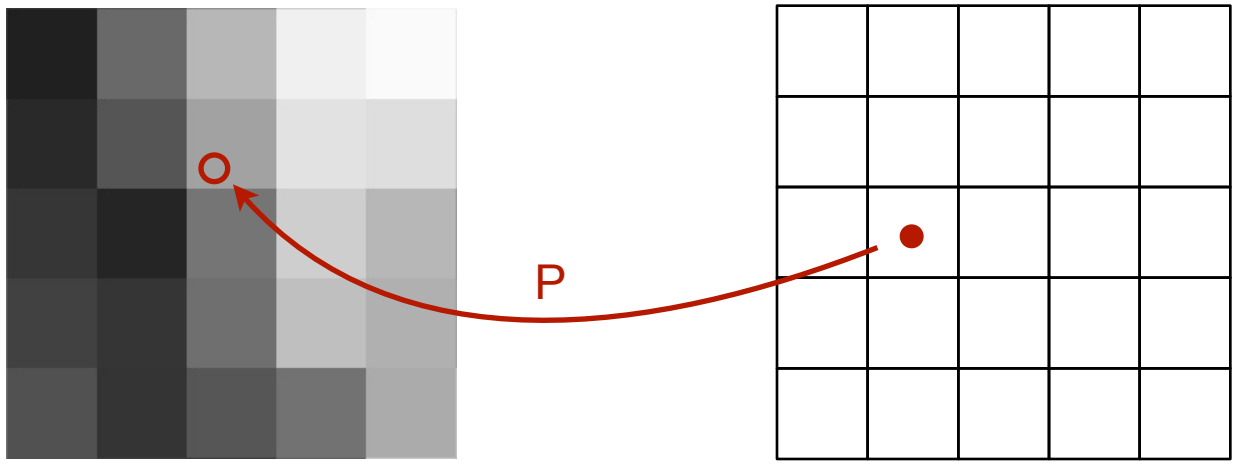

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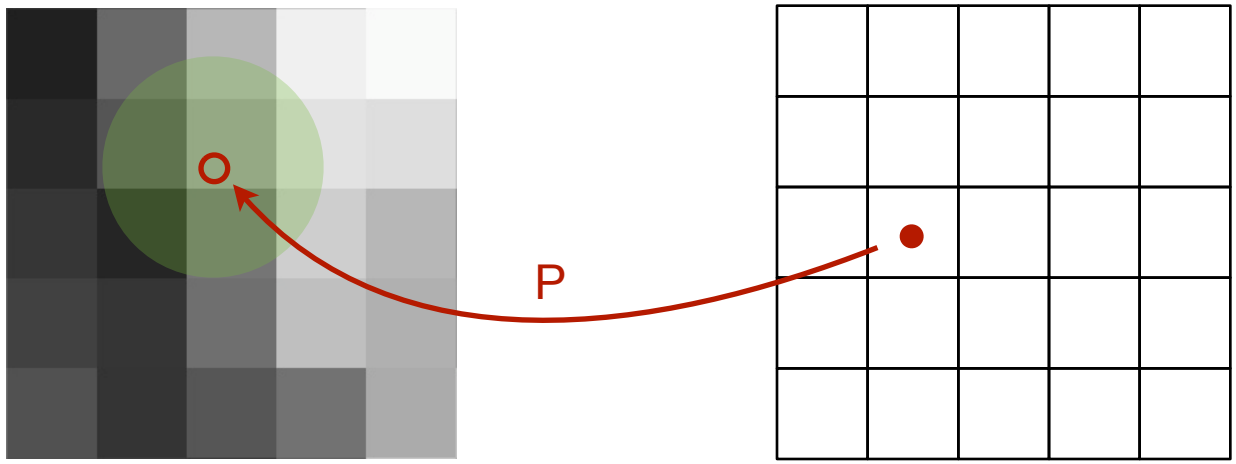

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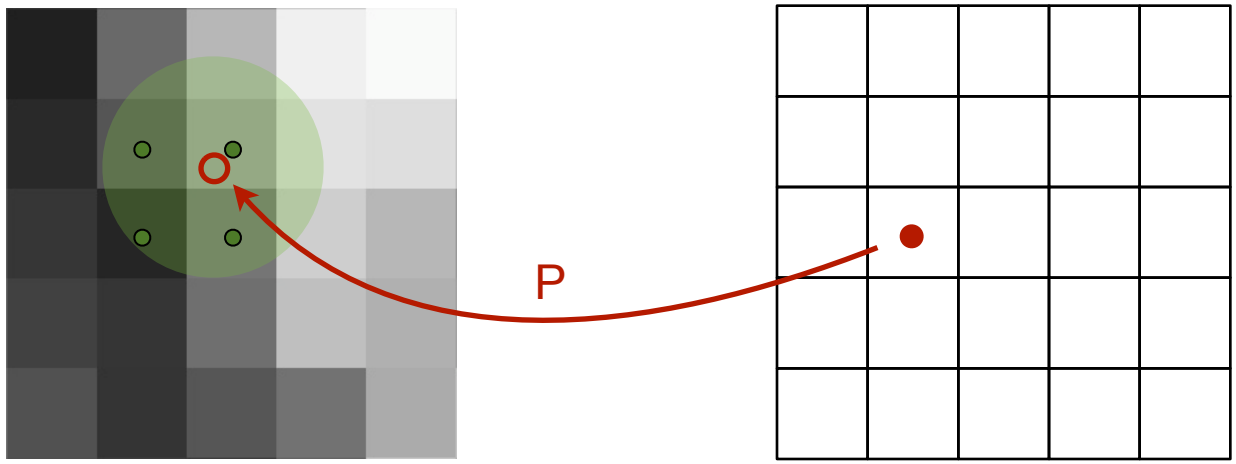

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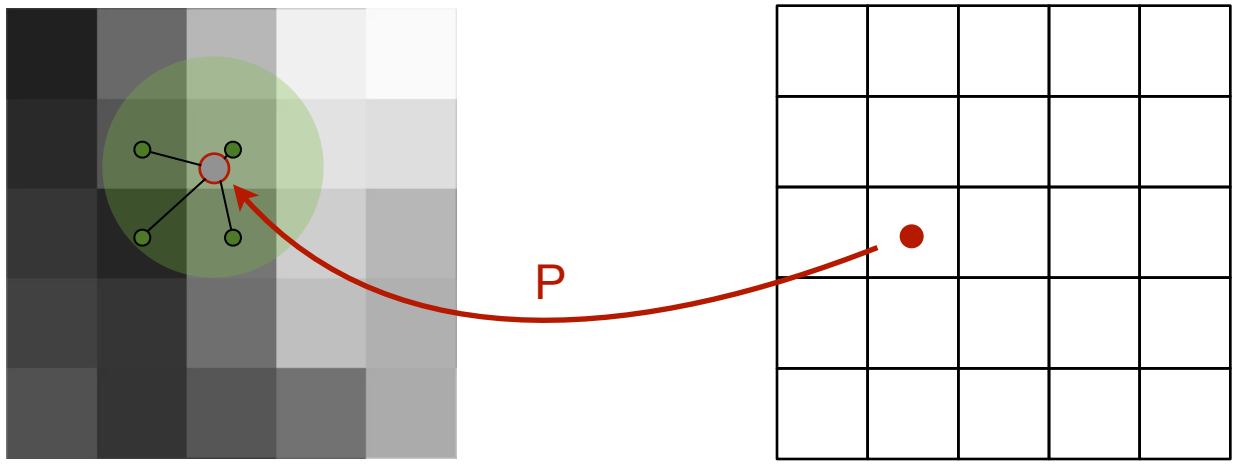

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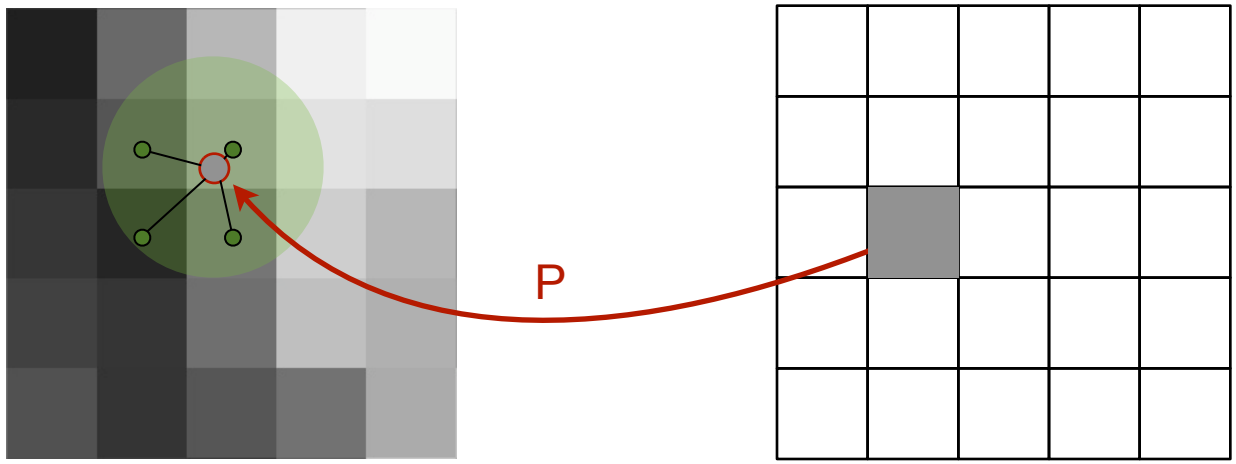
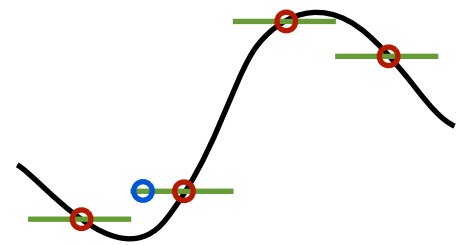
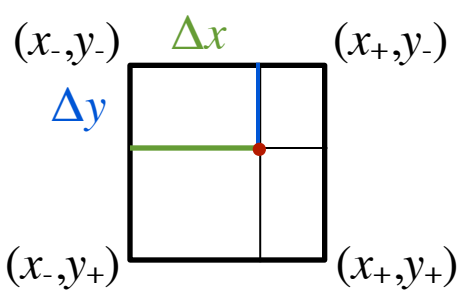


Resampling

- Intensity values off the pixel raster
  - nearest neighbor: value of nearest known point
  - original intensities are preserved
  - image structures are shifted → aliasing!





$$I(x,y) = \begin{cases} (\Delta x \leq 0.5) \wedge (\Delta y \leq 0.5): & I(x_-, y_-) \\ (\Delta x \leq 0.5) \wedge (\Delta y > 0.5): & I(x_-, y_+) \\ (\Delta x > 0.5) \wedge (\Delta y \leq 0.5): & I(x_+, y_-) \\ (\Delta x > 0.5) \wedge (\Delta y > 0.5): & I(x_+, y_+) \end{cases}$$

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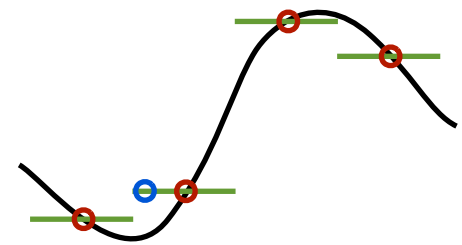

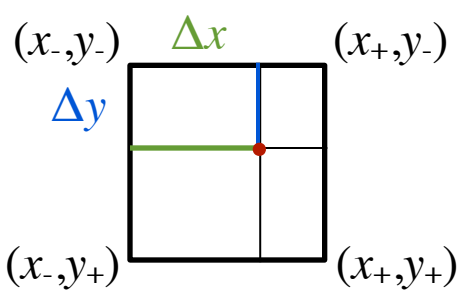

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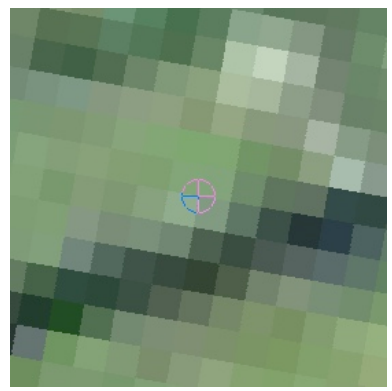
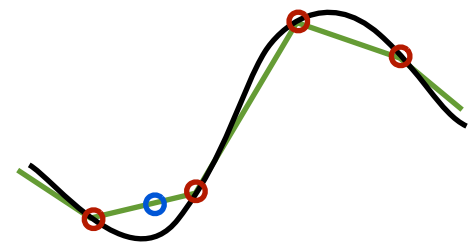
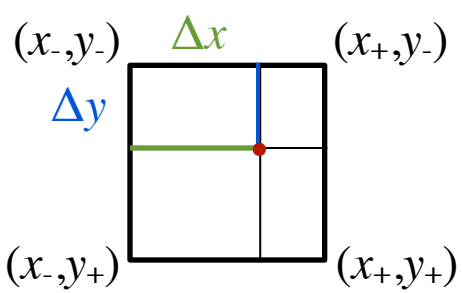
Resampling

- Intensity values off the pixel raster
  - bilinear interpolation in a (2 x 2) neighborhood
  - linear in each coordinate, but interpolant is 2nd order surface
  - noticeable smoothing







$$
\begin{array}{rcccccc}
I(x,y) & = & (1-\Delta x) & \cdot & (1-\Delta y) & \cdot & I(x_-,y_-) \quad + \\
 & & \Delta x & \cdot & (1-\Delta y) & \cdot & I(x_+,y_-) \quad + \\
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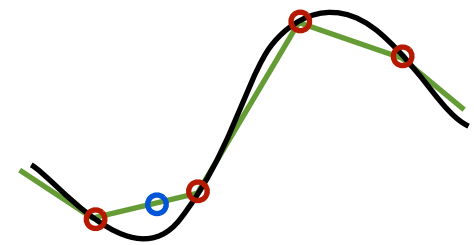

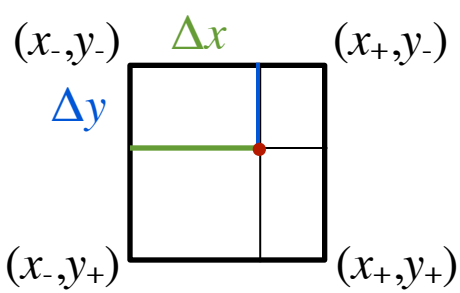

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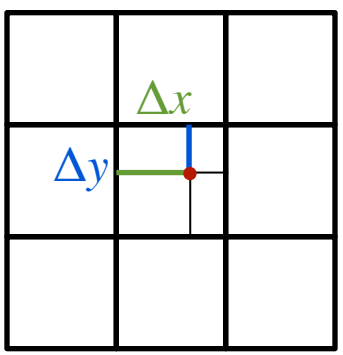
$$

Resampling

- Intensity values off the pixel raster
  - bicubic interpolation in a (4 x 4) neighborhood
  - cubic polynomial in each coordinate
  - theoretically still not a smooth interpolant, but in practice very good



$$I(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} \cdot (1+\Delta x)^i \cdot (1+\Delta y)^j$$

$$a_{ij} = f\big(I(x_{--}, y_{--}), I(x_{-}, y_{--}), \ldots, I(x_{++}, y_{++})\big)$$

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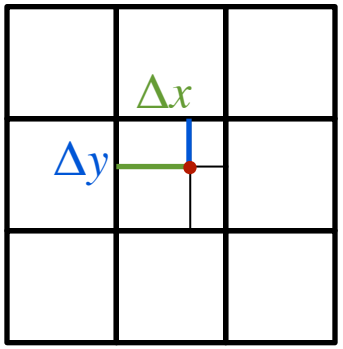

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Planar Rectification

- planar object surface
  - mapping from orthophoto plane to object plane is a parallel projection, thus **affine**

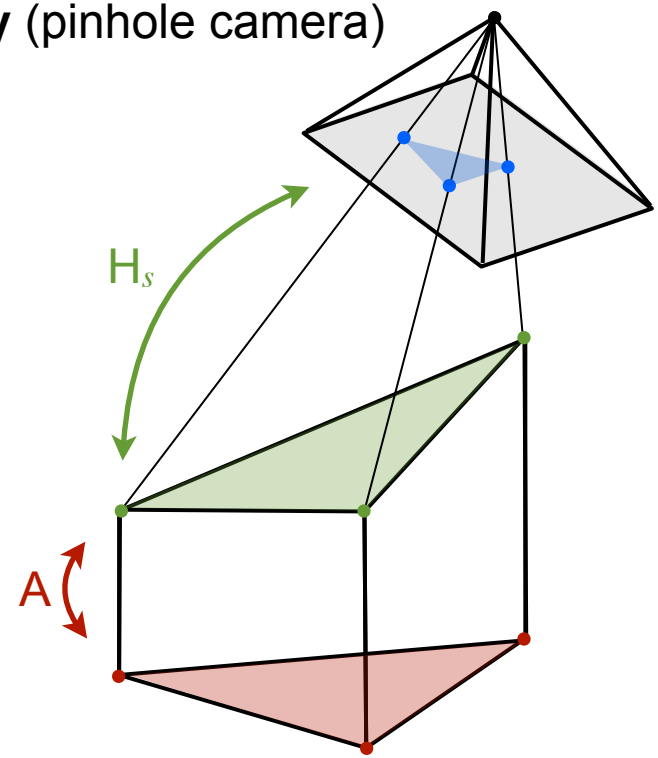
    6 parameters: position, orientation, anisotropic scaling and shear of coordinate axis
  - mapping between object plane and image plane is line-preserving and 2D-to-2D, so a **projectivity** (pinhole camera)

    8 parameters for homography
  - combination is also a **projectivity**

$$\mathbf{x}' = \mathsf{A}\mathsf{H}_s\mathbf{x} = \mathsf{H}\mathbf{x}$$

$$\mathsf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

$$\mathsf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$



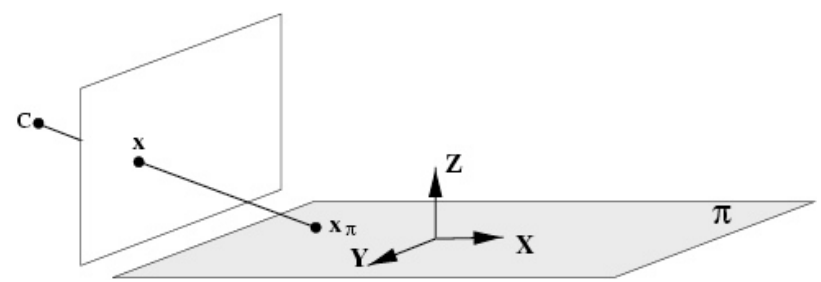
Planar Rectification

- Homography from orientation parameters
  - assumption: object in (X,Y)-plane (can always be reached by a Helmert transformation)

$$\mathbf{x} = \mathsf{PT}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \widetilde{\mathsf{P}}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \tilde{p}_{11} & \tilde{p}_{12} & \tilde{p}_{14} \\ \tilde{p}_{21} & \tilde{p}_{22} & \tilde{p}_{24} \\ \tilde{p}_{31} & \tilde{p}_{32} & \tilde{p}_{34} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mathsf{H} = \begin{bmatrix} \tilde{p}_{11} & \tilde{p}_{12} & \tilde{p}_{14} \\ \tilde{p}_{21} & \tilde{p}_{22} & \tilde{p}_{24} \\ \tilde{p}_{31} & \tilde{p}_{32} & \tilde{p}_{34} \end{bmatrix}^{-1}$$

$$\mathbf{x}' = \mathsf{H}\mathbf{x}$$

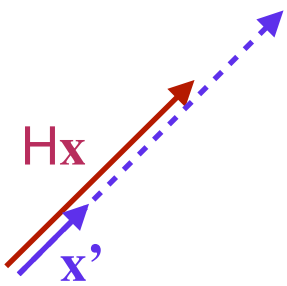


- ***Note***: other than perspective projection, the projectivity is one-to-one (bijective) and thus invertible

Planar Rectification

- direct estimation of the homography
  - all we need are the 8 unknowns of H
  - a pair of corresponding points yield 2 linearly independent equations
  - ≥4 correspondences (no 3 of them on a line) yield ≥8 equations, solution via singular value decomposition
  - neither the interior nor the exterior orientation need to be known!

$$\lambda \mathbf{x}' = \mathsf{H}\mathbf{x} \qquad \mathsf{H}\mathbf{x} \times \mathbf{x}' = \mathbf{0}$$



$$\begin{bmatrix} 0 & 0 & 0 & x & y & 1 & -xy' & -yy' & -y' \\ -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Planar Rectification

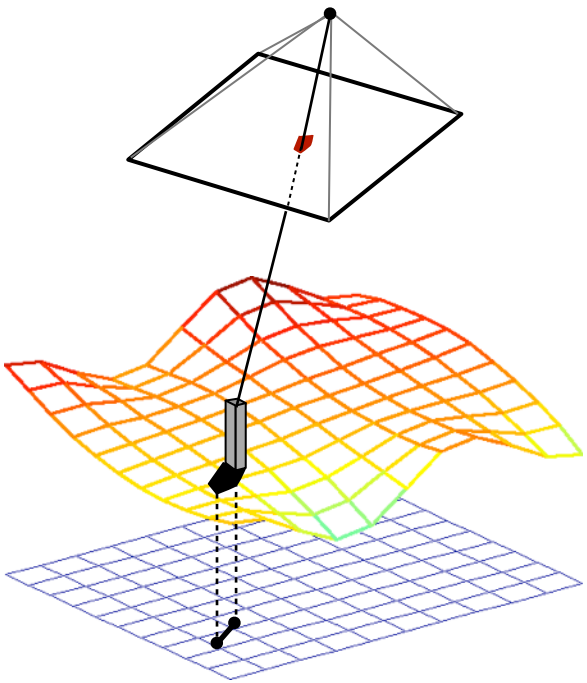
- Example: facade orthophoto

Orthophoto - Accuracy

- Two main sources of error

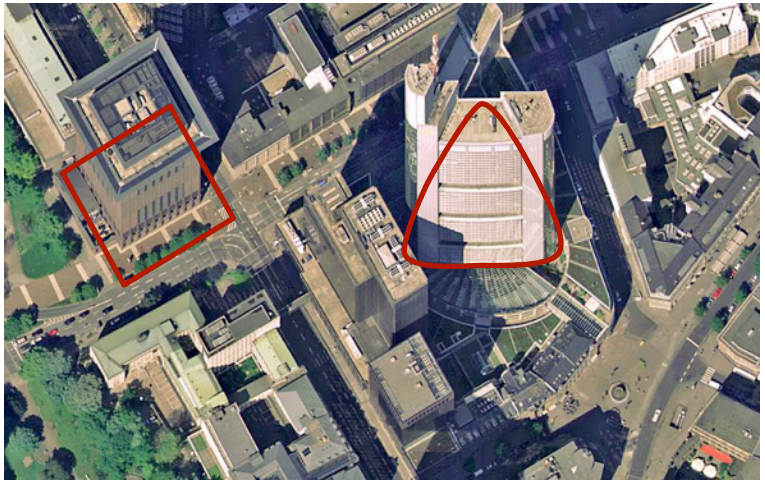
- inaccurate DSM
  - height errors in the surface model are mapped to planimetric errors in the orthophoto
  - points are displaced in radial direction w.r.t. the nadir

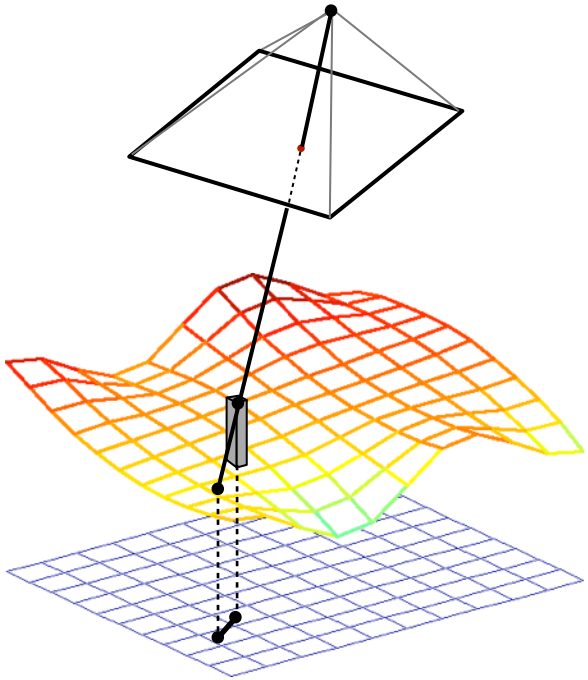
- Occlusions
  - string relief of the surface leads to occlusions
  - the intensities required for the orthophoto are not found in the original image



Orthophoto - Accuracy

- inaccurate DSM
  - often happens when of-the-shelf terrain models (DTMs) are used, especially if they do not contain buildings and vegetation
  - nowadays less frequent because of the increased availability of high-quality surface models (LiDAR, dense image matching) and 3D city models

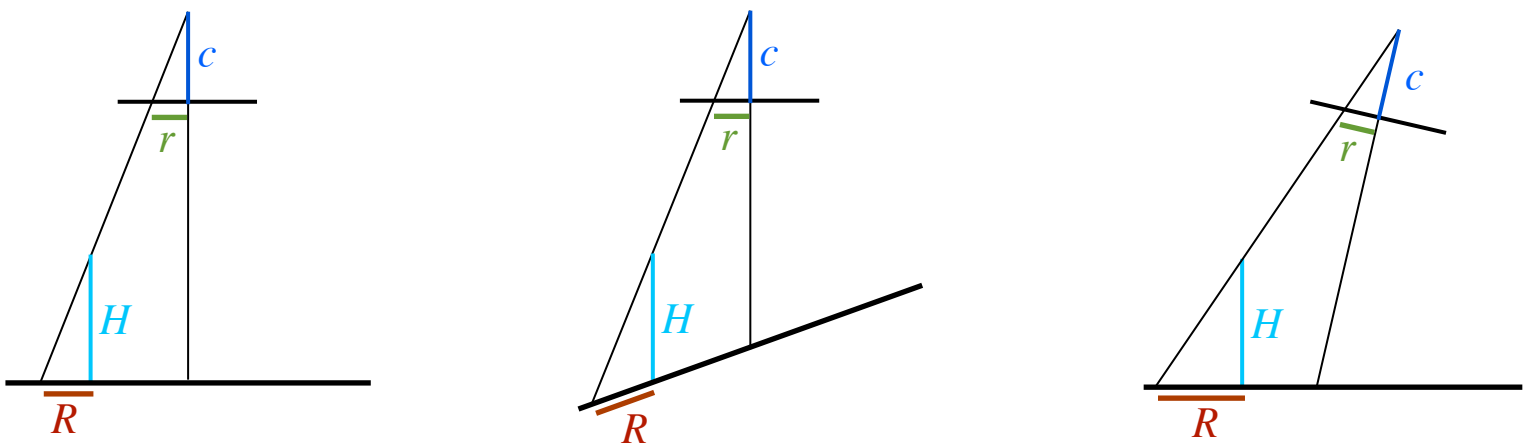




Orthophoto - Accuracy

- Radial displacement
  - Vertical errors in the surface model are mapped to radial planimetric errors
  - the error depends on
    - the magnitude of the height error
    - the orientation
    - the point's position in the image
    - the surface slope
  - for approximate nadir views and flat terrain

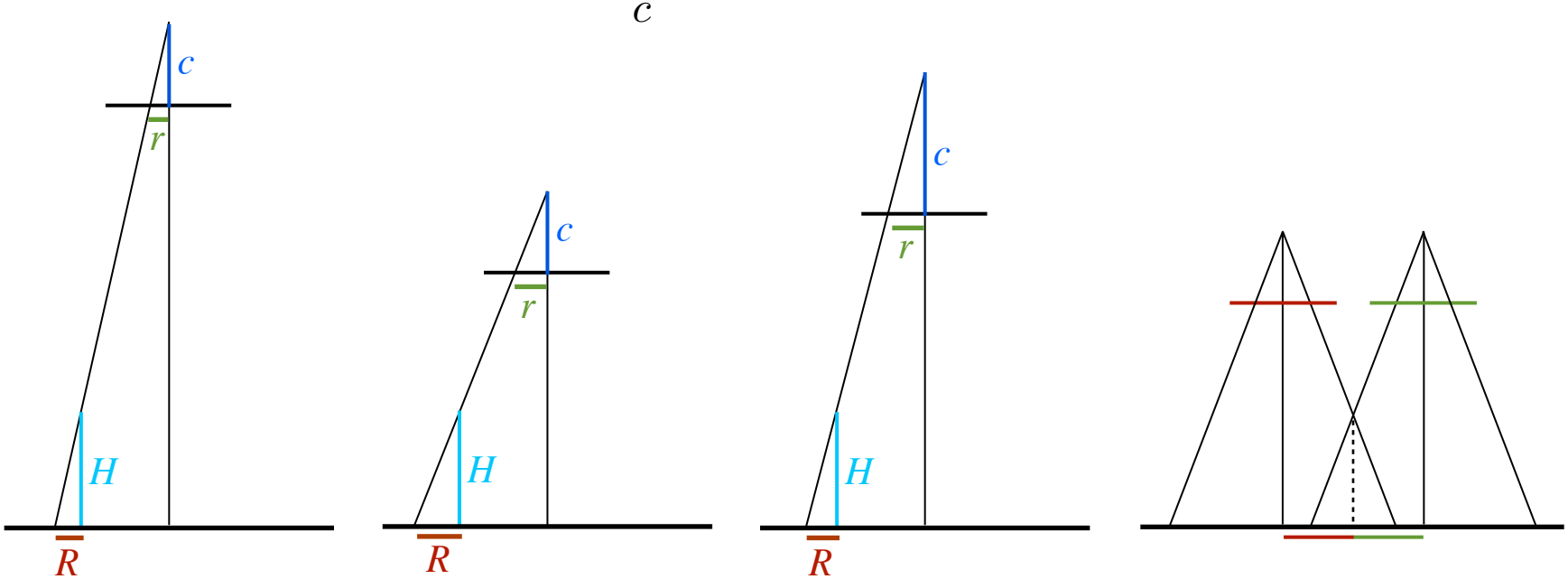
$$R = H \cdot \tan\beta = \frac{r}{c} \cdot H$$



Orthophoto - Accuracy

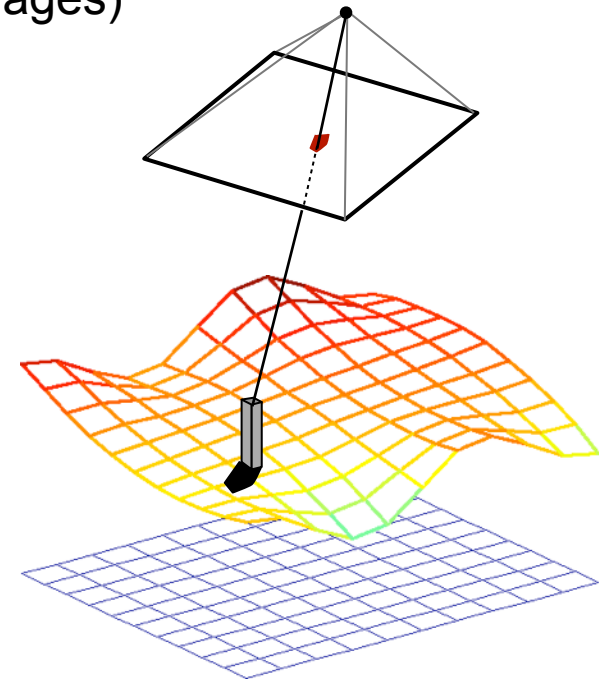
- Radial displacement increases with
  - increasing distance from the principal point / nadir (i.e. in image blocks also with decreasing overlap → border areas of images have to be used)
  - smaller camera constant resp. lower flying height

$$R = \frac{r}{c} \cdot H$$



Orthophoto - Accuracy

- Occlusions
  - strong relief leads to occlusions
  - brightness values for regions that are visible in the orthophoto (“from above”) are not observable in the original image
  - in weaker form: area is visible, but perspective foreshortening is too strong to extract usable texture (especially near the border of wide-angle images)



True Orthophoto

- Solution
  - Detection of occluded areas using the DSM
  - Use of multiple images to obtain an orthophoto without holes
  - often called “true orthophoto”