DEFORMATION MEASUREMENTS FROM AERIAL PHOTOGRAPHS
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ABSTRACT
Deformation measurements are essential for an extensive analysis and interpretation of the geometric behaviour of the deformable body. Aerial photographs are evaluated in monitoring ground displacements in a remote area with respect to different approaches, results and difficulties encountered. The use of analytical plotter and the implementation of the dynamic state of the area to strengthen the mathematical model (Kalman filtering) are discussed.

RESUME
Les mesures des déformations sont essentielles pour une analyse complète et pour l'interprétation du compartement géométrique des corps déformables. Les photographies aériennes sont évaluées comme contrôles des déplacements du sol pour une région donnée, pour différentes approches, les résultats et les difficultés rencontrés y sont présentés. On discute également de l'utilisation du restituteur analytique et de l'introduction de l'état dynamique de la région pour renforcer le modèle mathématique (filtre de Kalman).

ZUSAMMENFASSUNG
Für die Analyse und Interpretation des geometrischen Verhaltens eines verformbaren Körpers sind Deformationsmessungen äusserst wichtig. Der vorliegende Bericht befasst sich mit verschiedenen Verfahren der Luftbildüberwachung von Bodenbewegungen in einem abgelegenen Gebiet, ihren Ergebnissen sowie auftretenden Schwierigkeiten. Der Einsatz eines analytischen Auswertegerätes sowie die Einführung des dynamischen Zustands zur Stärkung des mathematischen Modells (Kalman Filtering) wird erläutert.

1. DEFORMATION SURVEYS AND PHOTOGRAMMETRY
Under the physical model known as 'cause' - 'transmission' - 'effect', almost every part of the earth's surface and even man-made structures are subject to variations of shape and position with time. The period of these changes differs from case to case depending on the individual characteristics of the deformable body. The determination, and sometimes also the interpretation of these movements is the objective of deformation surveys.

The study of deformations serves two essential purposes: First, it provides metric information on the physical behaviour of the deformable body (e.g., stability or rate of displacements). Secondly, it supports and aids in improving new design theories with respect to a better insight into the deformation mechanism.

For monitoring purposes, the entire deformable object is represented by a number of properly distributed discrete points, the so-called detail points. Consequently, a possible
deformation of the whole body must be estimated from the movements of these detail points. The connection of the detail points with each other or to 'reference' points located 'outside' of the deformable body introduces the concept of relative and absolute deformation measurements, respectively. Present methods for detecting and monitoring movements can be divided into two basic groups: physical and geometric [Chrzanowski, 1981]. Physical methods are used to measure relative displacements using various mechanical instruments such as tilters or strain gauges. On the other hand, geometric methods can be employed to monitor both relative and absolute movements. Depending on the extent of the deformation, the geometric techniques vary from surveying and photogrammetric methods to very long baseline radio interferometry and satellite laser ranging.
Surveying monitoring networks are generally one-, two-, or three-dimensional networks in Euclidean space. Although they can reach high accuracies, they are based solely on information obtained at specific preselected points, and thus, are point dependent. Also, the time required to perform a complete survey of the detail points may well be longer than the frequency of movements. This means that actual displacements may be unreliable or left undetected, thereby reducing the overall accuracy of the survey.
On the contrary, a photograph records the geometry of the object at the moment of exposure. This means that the deformable body can be captured at certain time intervals even during a dynamic process. In addition to its instantaneous character, a photograph represents a remote, complete and permanent record of the deformable object. If the displacements are two-dimensional, a single photograph provides a one-to-one correspondence between image and object coordinates through the projective relation between two planes. For three-dimensional movements, two or more photographs covering the object are required.
Aerial and terrestrial photogrammetric techniques have gained increasing popularity for monitoring deformations within the past fifteen years. Pertinent references are given in Armenakis (1983).

2. PRACTICAL APPLICATION; METHODS, RESULTS AND DIFFICULTIES
Aerial photographs were used to monitor ground movements in a remote area, described in Faig (1984). The displacements were determined either as differences of absolute coordinates among the different epochs or directly [ibid]. In this context three analytical approaches were evaluated, namely bundle adjustment with additional parameters, adjustment of independent models and direct object displacements from model coordinate-differences.
The displacements between the first and second epoch were determined using the bundle adjustment program GEBAT-V [El-Hakim, 1982] which incorporates additional parameters. Control points were kept the same for both epochs and the displacement vectors were calculated as differences of the two separate states. The estimation of the obtainable accuracies was based on the comparison of these results with those obtained by ground surveying techniques, and also on the evaluation of the photogrammetric results using one- and three-
dimensional tests [ibid].
For the third epoch the displacements were determined directly
from model coordinate differences because sufficient control
points were not available. The program PSUB (Photogrammetric
SUBsidence), [Armenakis, 1983], was used. Indication of the
achievable accuracies are given by mean values and RMS of:

a) photo-residuals after relative orientation:
\[ \bar{r} = 0.0 \, \mu m, \sigma_r = 2 \, \mu m \]

b) ground coordinate differences after absolute orientation of
model epoch 1:
\[ \bar{\delta}X = 0.20 \, \text{cm} \quad \bar{\delta}Y = -0.01 \, \text{cm} \quad \bar{\delta}Z = 0.00 \, \text{cm} \]
\[ \sigma_{\delta X} = \pm 22 \, \text{cm} \quad \sigma_{\delta Y} = \pm 8 \, \text{cm} \quad \sigma_{\delta Z} = \pm 7 \, \text{cm} \]

c) subsidence differences between surveying and photogrammetric
results:
\[ \bar{\Delta}Z = 10.5 \, \text{cm} \quad \sigma_{\Delta Z} = \pm 7 \, \text{cm} \]

The relative accuracy of this approach estimated from c) and
from the camera-to-object distance was approximately 1/14,000.
The average photo-scale was about 1/7000, varying of course
significantly, due to the large elevation differences of more
than 500 m.

Finally another test was performed in a neighboring area using
the principle of aerotriangulation. Using the existing control
of epoch \( t_k \) (\( t_j < t_k \)), coordinates of common stable points
between periods \( t_j \) and \( t_k \) were computed by the analytical
independent model approach. The program PAT-M43 [Ackermann et
al., 1973], was used for both epochs with the following input:
1) model coordinates formed analytically and ii) the common
points as control. Since control points were available only at
time \( t_k \) (\( t_i < t_k \)) the program PSUB was used as well. The
objective of the test was to evaluate the performance of the
latter program. The differences in the final results were
compared and gave the following mean values and standard
deviations respectively:
\[ \bar{\delta}X = -0.2 \, \text{cm} \quad \bar{\delta}Y = 1.5 \, \text{cm} \quad \bar{\delta}Z = -1.0 \, \text{cm} \]
\[ \sigma_{\delta X} = \pm 1.6 \, \text{cm} \quad \sigma_{\delta Y} = \pm 2 \, \text{cm} \quad \sigma_{\delta Z} = \pm 3.5 \, \text{cm} \]

These results indicate that in cases where highly sophisticated
photogrammetric software is not available, simpler programs can
provide the means for carrying out deformation projects by
photogrammetry.
The photogrammetric potential in the field of deformation
studies has not yet been fully exploited. The following
recommendations were adopted at the III International Symposium
on Deformation Measurements by Geodetic Methods [Budapest,
1982]: "In accordance with the resolutions of the Montreux FIG
Congress, more attention must be paid to the application of
remote sensing technology in the field of deformation
measurements in cooperation with the ISPRS Commission 5. It is
justified to increase research in complex photogrammetric systems for use in deformation measurements. The international cooperation, connected with these complex systems, should be expanded."

Until now, most of the deformation models were of a static type aiming for a statistical statement showing the existence or non-existence of movements in the space domain. Present developments focus on dynamic models where deformations are studied with respect to time and frequency of occurrence and moreover, as functions of the causative parameters [Welsch, 1981].

As a direct three-dimensional monitoring scheme, photogrammetric techniques have not used the time factor extensively as the fourth dimension. To the best of the author's knowledge, monitoring photogrammetry has merely been used as a single frame in a movie film, one out of a series of frames which together present the complete four-dimensional picture of the moving object [Andrei and Cotovanu, 1978]. That is, each new position of the points determining the trajectory of the body in motion is estimated independently to the preceding one. What is needed then, is a connecting factor for all the single frames. The time can serve for this purpose if it can be expressed as parameter of functions representing a dynamic state.

Another subject to be considered is the need to make observations on different photographs taken at different epochs. Their quality, the elimination of possible point-misidentifications and the ease of their storage and retrieval are of major importance for a modern photogrammetric monitoring system. These aspects can be investigated with respect to sophisticated analytical plotters.

Finally attention should be paid in the quality and existence of stable points since they comprise the reference frame of our work.

3. REAL-TIME PHOTO-OBSERVATIONS AND DISPLACEMENTS USING THE ANALYTICAL PLOTTER

For the photogrammetric determination of displacements, multiple observation epochs are necessary. Therefore, measurements must be performed on the same clearly identified photo-images of discrete points, where artificially targetted points would be the ideal situation. However, in remote areas it is not feasible to keep targets in place over long periods of time and their installation is a tedious operation, not to mention that movements can occur at areas not covered by targets. On the other hand, selection of natural points offers more flexibility as it circumvents these difficulties.

The identification of physical points sometimes is laborious as their appearance in different photographs is radically different due to changes in perspective. To overcome this problem, a 'cross-identification' procedure for the photo-measurements has been proposed [Armenakis, 1983]. This method employs a point transfer and marking instrument for locating the points as well as a stereocomparator for measuring image-coordinates. The preservation of stereoscopic perception in both operations provides better accuracies in locating the points of interest [Eden, 1973]. However, large differences between photo-scales result in poor stereoscopy during point
identification when optical image rotation and zoom magnification are not incorporated. Also, the storage of photo-coordinates is not efficient. Consequently, a solution is being sought that could combine the aforementioned operations, ideally. Point identification, image-measurements, as well as storage of photo-coordinates, retrieval, and editing would be performed 'simultaneously' using an analytical formulation relating the images of the same point on different photographs at different time intervals. Software is presently being developed to perform this task on the Analytical Plotter O.M.I.-AP/2C.

Considering the point identification problem using the 'cross-identification' method, a solution needs to be: i) independent of the absolute attitude of the camera stations, and ii) independent of the object space. Consequently, what is sought can be defined as: Given a photograph L1, and a photo-vector of a point at epoch \( t_1 \), determine the corresponding photo-vector on another photograph R2 at epoch \( t_2 \). The proposed method, (Fig. 1), employs formation of 'real' and 'fictitious' models based on the relative position of photographs of the same or different times respectively.

![Diagram](image-url)

**Figure 1.** Determination of corresponding image-coordinates from a reference photograph (i.e., L1).

1a - actual situation (real models L1, R1 and L2, R2)
1b - non-intersection of corresponding photo-vectors
1c - fictitious model formation (L1, R2)

Relative positions of the photographs can be established via dependent pair analytical relative orientation. Using the analytical plotter in a stereo-comparator mode, the coordinates of at least five pairs of conjugate points have to be measured. The attitude \( (\omega, \phi, k) \), by, bz) of the dependent photograph R2 with respect to the fixed one is estimated using the coplanarity equations. Instead of positioning the image coordinates on both photographs through the collinearity equations as functions of: a) the \( X_m, Y_m, Z_m \) (machine coordinates), b) attitudes of both cameras, and c) interior orientation elements, a modified real-time positioning program [Kratky, 1980] can now be used. This program maintains the stereo-comparator mode by transferring \( X_m, Y_m \) values directly
to image-coordinates of the fixed photograph L1, and can serve two purposes. First, the operator selects well identified detail points at the initial stage of the 'cross-identification' method. Secondly, after the image-coordinates have been recorded, their relocation through the servo-motors is feasible. Entering then the \( Z_m \) value from the footwheel, a corresponding model position \( X_M', Y_M' \) is computed. From the model position \( (X_M', Y_M', Z_m) \) and the attitude of the dependent photograph, R2, the corresponding image coordinates on R2 are computed using the collinearity equations. Figure 2 illustrates the real-time positioning procedure.

The formation of the 'fictitious' model between photographs L1 and R2 provides the means for measuring and storing the photo-coordinates of the detail points on photographs (L1 and R2) at different epochs. Photograph R1 now replaces R2 on the right photo-carrier and is relatively oriented with respect to photograph L1. Recalling the recorded image coordinates of the selected points on photo L1 and using the real-time positioning program, their 'real' model coordinates (in epoch 1) and their conjugate image-coordinates on photo R1 are determined and recorded. A similar procedure follows for photographs L2 and R2. They are now placed on the left and right photo-carriers, respectively. However, this time the procedure works in reverse. The reference or fixed photograph, set on the right carrier, is photo R2 because the image coordinates of the selected points have been recorded from it during the cross-identification stage ('fictitious' model) and are recoverable. Consequently, a 'real' model (L2, R2) is formed, resulting in 'real' model coordinates and conjugate image-coordinates (on photo L2) of the detail points.

![Diagram](https://via.placeholder.com/150)

**Figure 2.** Identification of detail points using a real-time positioning procedure.

At this point the necessary information required to determine displacements has been gathered.

Then displacements can be computed from differences between absolute coordinates. To reduce the effect of a 'weak' solution for absolute orientation (e.g., insufficient or poor ground control) another approach is chosen. The displacements
are determined directly from model coordinate differences (Faig, 1984). A knowledge of stable points, located 'outside' of the deformable body, is required (non-existence of reference points is discussed in section 4).

As previously mentioned, a large K rotation and different photo-scales lead to poor stereoscopic perception. The large K rotation between two photographs can be avoided either by eliminating it when the photographs are placed onto the photo-carriages or by rotating the images with the help of Dove prisms under computer control. Various object-to-camera distances and tilts between two photographs result in different photo-scales and therefore changes in magnification. These changes can be computed such that the magnification values for each photograph are updated. The continuous (zoom) magnification is adjusted in the optical train by step motors under computer control [Masry and Faig, 1977].

4. DYNAMIC STATE OF DISPLACEMENTS

A detail point on the deformable body can be considered as changing position in time with respect to a reference system, if displacement occurs. If successive measurements are performed, new data become available. Consequently, there are two time dependent factors; the actual position of the state-space vector which changes progressively as a result of a cause, and the new observations at different time instants. The variation of the position parameters as well as the observations with respect to time is a case known as Kalman filtering [Vanicek and Krakliwsky, 1982]. We have, therefore, two discrete groups of equations in Kalman filtering: the prediction (or dynamic model) equations and the update (or observation) equations. While the observation equations can be manipulated by standard procedures in least squares, the transition function characterizing the state of the dynamic model from time $t_{i-1}$ to $t_i$ is difficult to define. If the moving body does not follow a certain physical law, predictions of the state between the periods of measurements, or even at any time $t_i$, are not possible without many hypotheses.

Given these considerations, an alternative approach was sought to express the time-dependent phenomenon of deformations. Instead of modelling the laws governing the motions of the deformable body, its kinematic behaviour could be considered. Thus the time can be expressed as parameter of velocity and acceleration. Under this concept, the detail points representing the object are in a state of continuous motion with respect to a certain reference system.

Therefore, instead of studying relative positions of the points at different time intervals we could estimate their relative velocities and accelerations [Papo and Perelmutter, 1982]. If a set of points is defined in the four-dimensional domain then the coordinates of a point $p_i$ can be expressed as functions of time. In vector form this is written as:

$$
\mathbf{r}_i(t_j) = [x_i(t_j), y_i(t_j), z_i(t_j)]^T, \quad i=\text{point } i, \quad j=\text{time } j
$$

(1)

If the motion of the points is expressed in linear form by series expansion, this leads to:
\[ \tilde{r}_i(t_k) - \tilde{r}_i(t_j) = (\Delta x_i(\Delta t_{jk}) \Delta y_i(\Delta t_{jk}) \Delta z_i(\Delta t_{jk}))^T \]
\[ = (dx/dt \ dy/dt \ dz/dt)^T (t_k-t_j), \ (t_j < t_k) \] (2)

Therefore, the three velocity components determine the deformation of a body at different time intervals.

Since displacements of a point can be expressed as functions of its velocity components with respect to a reference system, one way to determine these components is to include them in the photogrammetric functional model as unknown parameters. The photogrammetric model will comprise the extended collinearity equations. The functional form of the bundle adjustment with self-calibration, including the kinematic characteristics at epoch \( t_j \) is:

\[ F(X_1, X_2, \dot{X}_3) = L \] (3)

and in linearized form:

\[ A_{1j}X_1 + A_{2j}X_2 + A_{3j}(t_j-t_0)\dot{X}_3 = L + v \] (4)

where:
- \( X_1 \): vector of corrections to the approximate parameters related to the exterior orientation and additional parameters;
- \( X_2 \): vector of corrections to the approximate object coordinates;
- \( X_3 \): vector of corrections to the approximate velocity parameters;
- \( L \): vector of photo-observations;
- \( v \): vector of residuals;
- \( A_{1j}, A_{2j}, A_{3j} \): are the corresponding configuration matrices.

By repeating the photogrammetric observations at different epochs the system of equations (4) becomes:

\[
\begin{bmatrix}
A_{1j} & A_{2j} & A_{3j}(t_j-t_0) \\
A_{1k} & A_{2k} & A_{3k}(t_k-t_0) \\
\vdots & \vdots & \vdots \\
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
\dot{X}_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
L_j \\
L_k \\
\vdots \\
\end{bmatrix}
+ 
\begin{bmatrix}
v_j \\
v_k \\
\vdots \\
\end{bmatrix}
\] (5)

It should be noted at this point that the system of equations (4), (and therefore of equations (5)), is rank deficient unless a zero-variance computational base, [Niemeyer, 1982], is defined. One way of solving a singular problem is by imposing a particular set of constraints [Vanicek and Krakiwsky, 1982], such as: introducing reference points and setting their velocities equal to zero (e.g., as pseudo heavy weighted parameters). Another method is the use of inner-constraints which are a type of minimal constraints [Vanicek and Krakiwsky, 1982; Fraser and Gruendig, 1984; Papo and Perelmuter, 1982]. For aerial photography constraints can be applied merely on the \( X_2 \) and \( \dot{X}_3 \) parameters, respectively.

5. CONCLUDING REMARKS

The intended monitoring scheme to determine deformations must have the following characteristics [Chen, 1983]: a) Higher
accuracy requirements, b) Repeatability of observations, c) Integrations of different types of observations, d) Sophisticated analysis of the acquired data, and e) Requirement of interdisciplinary knowledge. The design of the monitoring scheme consists of the type and amount of data to be collected, their accuracies and the required specifications.

Work is presently in progress to include the time factor in the design of a photogrammetric monitoring scheme for deformations. Considerations are given regarding the operations of data acquisition and manipulation, as well as the formulation of the model.

The inclusion of the analytical stereoplotter as a tool for the photo-measurements is crucial. The expected accuracies are in the order of 3-5 μm in photo-scale. The analytical real-time positioning of the corresponding natural (or targetted) points in different epochs provides flexibility, elimination of identification errors, and easy retrieval and editing of the image coordinates. The image motion effect might somewhat reduce the quality of measurements as it causes blurred images which make precise measurements difficult.

With respect to the time component, the detail points are considered to be in a continuous motion. Although such phenomena can be investigated using Kalman filtering, possible lack of information regarding the direction and magnitude of displacements makes the formulation of the transition matrix difficult. Since displacements can be expressed as functions of time intervals and velocities, time is implicitly encompassed if the velocities are included as unknown parameters in the model. Under this alternative the time-dependent procedure is preserved because velocity links two different states.

Recapitulating, it is believed that the advantageous properties of photogrammetric capture of information, the incorporation of the analytical plotter, and the introduction of the time factor to a rigorous analytical photogrammetric model, can contribute to the field of deformation studies.

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